

A few electrons per ion scenario for the $B = 0$ metal-insulator transition in two dimensions

T.M. Klapwijk *

Center for Quantized Electronic Structures (QUEST) and Institute for Theoretical Physics, University of California at Santa Barbara, California 93106-4060

and

Department of Applied Physics and Materials Science Centre, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

S. Das Sarma

Institute for Theoretical Physics, University of California at Santa Barbara, California 93106-4060

and

Department of Physics, University of Maryland, College Park, Maryland 20742-4111

(February 1, 2008)

We argue on the basis of experimental numbers that the $B = 0$ metal-insulator transition in two dimensions, observed in Si-MOSFETs and in other two-dimensional systems, is likely to be due to a few strongly interacting electrons, which also interact strongly with the random positively ionized impurities. At the insulating side the electrons are all bound in pairs to the ions. On the metallic side free electrons exist which are scattered by ions dressed with electron-pairs and therefore alter the bare scattering potential of the ions. The physics at the metallic side of the transition is argued to be controlled by the classical to quantum transport cross-over leading to the observed non-monotonous dependence of the resistivity on temperature. This few electrons per ion scenario appears to be an experimentally realistic and testable scenario, which can also serve as a starting point for further theoretical analysis of the two-dimensional metal-insulator transition.

Introduction

A number of recent¹⁻⁵ striking⁶ experimental observations on the low temperature ($0.1 - 10\text{ K}$) transport properties of low density two dimensional electron/hole systems (2DES) have been interpreted on the basis of 2D metal-insulator transition⁶ (2D M-I-T), which is nominally prohibited by weak localization theories⁷. A critical density n_c ($\lesssim 10^{11}\text{ cm}^{-2}$) for Si-MOSFETs separates the "metallic" and the "insulating" sides of the transition with the resistivity showing a strong exponential (positive) temperature dependence in the insulating side and a weak negative temperature dependence (at least at lower temperatures) in the metallic side. Some of the published data show a remarkable scaling behavior in carrier density (n_s) and temperature (T), which

is consistent with a M-I-T $T = 0$ quantum phase transition phenomenology^{8,9}, and therefore a great deal of theoretical attention has been focused on this "2D M-I-T" phenomenon, with theoretical proposals¹⁰⁻¹⁵ ranging from the phenomenon being a superconductor-to-insulator transition to non-Fermi liquid theories. In addition, a more conventional explanation has been proposed by Alsthuler and Maslov¹⁶ based on charge traps in the silicon-dioxide.

In this article we provide a critical phenomenological discussion of the experimental system, concentrating almost entirely on the Si/SiO_2 MOSFET systems used by Kravchenko and collaborators¹⁻³ in their pioneering work. For this system a large body of detailed experimental published data is available, and the most compelling evidence for the existence of a 2D M-I-T has been established in this system. We believe that a detailed qualitative (perhaps even a semi-quantitative) understanding of the temperature dependent resistivity, $\rho(T)$, around the critical density can be developed for Si-MOSFETs on the basis of the considerations proposed here, in particular by focusing on the experimental parameters of Si-MOSFETs. Our physical considerations described below should serve to provide rather stringent experimental constraints for possible theories of the 2D M-I-T. We also provide a physically motivated conjecture about the mechanism that drives the transition in these systems. Our proposed 2D M-I-T scenario is precise and directly experimentally testable.

Mobility as a function of electron density

The mobility of MOSFETs, which can be defined only on the metallic side of the transition, $n_s > n_c$, is limited by scattering from surface roughness and ionized impurities¹⁷. In high-mobility samples¹⁸⁻²², such as used to measure the metal-insulator transition, high resistivity Si material is used, typical resistivities in excess of $100\Omega\text{ cm}$ or the density of donors less than 10^{14} cm^{-3} ,

*Correspondence should be addressed to T.M.K (e-mail: klapwijk@phys.rug.nl)

and therefore scattering by ionized donors can be ignored. The carrier density is varied by applying a voltage to the gate electrode which is separated from the silicon by insulating SiO_2 . At the interface a triangular potential well, "the inversion layer", is formed of which only the lowest subband is occupied at low temperatures. Experimentally, one observes studying conduction-processes parallel to the Si/SiO_2 interface, a non-monotonic dependence in the low temperature mobility as a function of density: for decreasing density an increase in mobility is seen reaching a maximum, called the peak-mobility, followed by a rapid decrease, as the carrier density is further reduced below n_{max} where the mobility reaches its maximum, μ_{max} .

It is understood that the decreasing and the increasing part of the mobility reflects scattering by different physical processes. Starting at high electron densities, high applied gate voltages, the carriers are pulled towards the Si/SiO_2 interface and experience therefore strongly the surface roughness. For high mobility MOSFETs such as used in these experiments¹⁸, the amplitude of the roughness is approximately 0.3 nm and the correlation length in the order of 7 nm . Upon lowering of the gate voltage the 2D gas moves away from the interface and the mobility increases. On the other hand the random ionized impurities, invariably present at the Si/SiO_2 interface as a result of the fabrication process, become more and more effective at lower electron densities, because the electronic screening is reduced. A high peak mobility and the associated low values of n_{max} at which the mobility peak occurs is taken as a signature¹⁸⁻²² of good quality MOSFETs because it signals both low surface roughness, which one can not do very much about, and also a low density of ionized impurities, which is controlled by working under very clean conditions in particular for the oxidation process.

The first point we make (which has not been emphasized in the existing theoretical literature on 2D M-I-T) is that the observed metal-insulator transition occurs at electron densities where the ionized impurity scattering dominates. In particular, the 2D M-I-T is only seen in samples with high μ_{max} , and usually with $n_c \ll n_{max}$. In our own samples²⁰ the metal-insulator transition occurs at carrier densities of about $8 \cdot 10^{10}\text{ cm}^{-2}$, far below the density n_{max} at which we have the peak mobility, $6 \cdot 10^{11}\text{ cm}^{-2}$. Consequently, we are in a regime where carrier scattering (and therefore transport) is dominated by ionized impurities. In this situation ($n_c \ll n_{max}$) electron densities are sufficiently low so that the screening of these ionized impurities has become strongly weakened. This has two implications. First, it means that the scattering potential should not be looked at as the surface roughness, namely a hard-walled potential landscape due to fixed fluctuations like a rocky landscape. The potential scattering is one due to ions, predominantly positive, located in laterally random positions, at the interface or slightly (only a few \AA) away from the interface inside the oxide. The second implication is that we have a system in which the electrons are facing random local-

ized attractors, *i.e.* the charged impurity ions, which are poorly screened. Thus, electronic screening is an important ingredient in understanding the observed 2D M-I-T. To the best of our knowledge a $n_c > n_{max}$ has never been reported in Si MOSFETs, samples with the highest peak-mobility¹ $\mu_{max} \sim 7 \cdot 10^4\text{ cm}^2/\text{Vs}$ have $n_c \leq n_{max}/2$, whereas samples with a more modest peak-mobility of $\sim 2 \cdot 10^4\text{ cm}^2/\text{Vs}$ typically have $n_c \leq n_{max}/5$.

How many electrons per ion?

It has been emphasized by Kravchenko and co-workers^{1,2,21} that to observe the 2D M-I-T the electron-electron interaction energy E_{e-e} should dominate over the Fermi-energy E_F , which means that $r_s = E_{e-e}/E_F = 1/(\pi n_s)^{1/2} a_B^*$ should be substantially larger than 1 (n_s the carrier density and a_B^* the Bohr radius taking into account the dielectric constant of the material). The M-I-T in MOSFETs is observed around $r_s \lesssim 10$. This argument has been used to justify the absence of a 2D M-I-T in an electron gas in GaAs/AlGaAs heterostructures. The lighter effective mass and the larger dielectric constant would push this regime to much lower densities. Convincing experimental support for this aspect of the scenario has been provided by the recent reports^{4,5} on a similar 2D M-I-T in a 2-dimensional hole gas in GaAs/AlGaAs heterostructures in which the holes have a very high effective mass. In these systems the transition occurs around $r_s \sim 18$. Moreover, very recently the metal-insulator transition has also been observed²³ in n-type GaAs for $r_s \sim 6$. Therefore we believe that there is convincing evidence that strong electron-electron interactions play a key role.

Nevertheless, although this low r_s (~ 10) regime can easily be reached in all Si-MOSFETs, the 2D M-I-T is only observed in high mobility MOSFETs^{1,24}. Apparently, some other materials parameters play a role in addition to the electron-electron interaction. As we emphasized above a high mobility means a relatively low density of ionized impurities at the Si/SiO_2 interface. What is the density of these ionized impurities in practice? One way to determine their density is by measuring the capacitance as a function of voltage¹⁷. Taking into account the materials parameters of the metal gate, the oxide, and the silicon we know what the ideal C,V-curve should look like (in the absence of random charged impurities). The observed shift from the theoretical values provides the density of ions in the oxide. A second method^{19,22} is to perform an accurate quantitative evaluation of the mobility *vs.* density curve, using the well-established theory for the scattering processes at densities around n_{max} . Both methods lead to a number of about $3 \cdot 10^{10}\text{ cm}^{-2}$ for high-mobility MOSFETs as used for the 2D M-I-T experiments. There is very little room to vary this number by more than a factor of 2. In fact it is roughly proportional to the peak mobility μ_{max} . A

decrease in the charged impurity density by a factor of 2 would raise the peak mobility¹⁹ from $25,000 \text{ cm}^2/\text{Vs}$ to $50,000 \text{ cm}^2/\text{Vs}$, which is not observed. In fact in our own samples¹⁸ we infer a charged impurity density of $2.4 \cdot 10^{10} \text{ cm}^{-2}$, which means that the average mutual separation is about 70 nm . Similar numbers are quoted by other researchers, in particular by Pudalov et al²¹. (Incidentally these numbers are much lower than those assumed in a recent manuscript of Altshuler and Maslov¹⁶, which tries to explain the 2D M-I-T as due trapping of electrons by ions, using a different mechanism than proposed by us.)

Experimentally the 2D M-I-T is observed²⁰ at a critical electron density n_c of around $8 \cdot 10^{10} \text{ cm}^{-2}$. Comparing with the density of ions we arrive at the *striking* result that at the critical density we have only 2 to 4 electrons per ion. This points to a natural explanation for the requirement of high mobility MOSFETs. Accepting the fact that the density should be low enough to have strong electron-electron interactions, if the mobility is lower by a factor of 3, for example below $10,000 \text{ cm}^2/\text{Vs}$, we have a 3 times higher density of ionic impurities and hence we will at a $r_s = 8$ always have a situation of less than one electron per impurity ion at the interface. Also, if we now compare the range of electron densities at which the anomalous metallic state is observed down to below the critical density we observe that all the action takes place in a regime in which the number of electrons decreases from at most 5 electrons/ion to 2 electrons/ion. We believe that transport physics at such a high density (2-5 electrons per ion) of random charged impurity centers is fundamentally different from high density electron systems because screening becomes strongly nonlinearly affected by ionic scattering and binding, leading to a sharp M-I-T at n_c when effectively all the electrons become bound to or trapped at individual ions at low enough temperatures.

Proposed scenario

We propose therefore that the 2D M-I-T is a phenomenon occurring in a system of interacting electrons and random positive ions with a density ratio of at least 2 electrons per ion at the critical density n_c . From the experiments carried out by Simonian et al²⁵ it is clear that the conducting (and insulating²⁶) states are sensitive to a parallel magnetic field which means that the spins play a crucial role. This observation points to the possible importance of spin-singlets. Taking these numbers into account and the possibility of spin singlets we arrive at the following conjecture. We suppose that we can divide the electrons in two groups: localized (bound) and delocalized (free) electrons. The localized electrons consist of two (or perhaps four) electrons per ion, bound with opposite spins at individual random ionic impurities. This binding is weak, as the binding energy²⁷ at low

electron density is a fraction of an meV . The delocalized electrons are scattered by impurity ions which are now dressed by two (or four) bound electrons which leads to a weaker scattering potential than the bare ion-potential. Hence starting from the higher electron density, metallic, side the scenario is that we have free electrons scattered of hard-walled ionic potentials. Then a crossover takes place as the electron density is lowered, which involves two processes. Electrons are scattered more strongly by the ions because screening weakens as impurity scattering induced level broadening modifies screening at low density. In addition, the number of free or unbound electrons decreases as more electrons bind to the individual ions with decreasing n_s . Upon further lowering the electron density it becomes possible that two (or four) electrons are trapped at the ions, which alters drastically the ionic scattering potential. This feedback mechanism continues (electrons getting bound to ions) as the electron density is lowered (and screening is weakened) until at $n_s = n_c$ suddenly all the electrons bind to ions observed as the 2D M-I-T. Clearly this would mean that the metallic conductance at the critical density is determined by a few electrons per square, a 'critical resistivity' of a few times h/e^2 as is experimentally observed^{1,20}.

Once all the electrons bind to ions the system at $T = 0$ is an insulator, but at finite temperature shows a finite conductivity either due to thermal unbinding or variable range hopping. The strong electron-electron interaction further helps the trapping at individual ions by keeping the electrons apart so as to minimize their Coulomb potential energy at a cost of their kinetic energy. In this sense our insulating 2D system might be considered a strongly pinned Wigner glass^{14,28}, *i.e.* an electron crystal strongly modified by the random charged impurity centers which act as the nucleation centers for localizing electrons. We believe that because of this few electrons per ion (FELPI-) scenario proposed here, it is more natural to think of the insulator as a system with few (2 to 4) electrons trapped or bound at charged impurity sites (with strong intersite Coulomb correlations), which is stabilized by gain in potential energy due to the ionic bonding. It is well known²⁹ that in a purely 2D system without the charged impurity centers Wigner crystallization occurs at a much lower density ($r_s \sim 38$). The observed 2D M-I-T in our scenario is the transition to this few electrons per ion (FELPI) insulating state.

In order to specify more precisely the experimental system we point out that the ions are located at the Si/SiO_2 interface, whereas the center of the 2-dimensional electron gas is, at the critical density n_c , at a distance of 7 nm away from this interface. Therefore the problem of individual ions with 2 electrons attached to them is reminiscent of that of the so-called D^- centers in $\text{GaAs}/\text{AlGaAs}$ ³⁰. These have been shown to exhibit a subtle dependence on the magnetic field including the so-called magnetic field induced dissociation, although the mutual interaction of the ion-electron system has not been studied yet. We also point out that the average

distance between the electrons at the transition is about 33 nm , whereas the Fermi wavelength is 120 nm . Finally, we point out that extracting the scattering time naively from the mobility around n_c leads to unrealistically small values of 0.1 ps , corresponding to a level broadening of 3 meV , in clear disagreement with the values obtained from Shubnikov-De Haas oscillations²⁶ which point to a level broadening of the order 0.1 meV . The latter value corresponds quite well to taking a mean free path of the order of the average inter-ionic distance and the appropriate Fermi velocity of $3 \cdot 10^6\text{ cm/s}$.

We now consider what happens within the FELPI-scenario if one applies an external electric or magnetic field^{3,20,25}. On the insulating side one would find an increase in conductance for a higher electric field in the usual way due to the electric field assisted hopping of the bound electrons. In this regime the application of a magnetic field eliminates (by lifting the spin degeneracy) the condition for pair binding, and we get more strongly bound electrons, of one per ion. This increased binding arises from the singlet-triplet bound state energy difference. So the application of a magnetic field on the insulating side leads to an increase in resistance. On the metallic side an electric field will strip the ions of bound electrons altering the scattering potential in making it a stronger scatterer and therefore an increase in resistance too. In addition, screening decreases (by a factor of two in 2D) due to the lifting of the spin-degeneracy leading to enhanced scattering and a consequent increase in resistance. The charged scattering centers behave somewhat like negative U-centers³¹ and the external magnetic field strongly affects the binding energies leading to the observed effects. As is evident from the magnetic field dependence of the D^- centers in GaAs/AlGaAs, the actual details may depend on the model³⁰.

Quantum to classical crossover

We now consider as an example³² a direct consequence of our observation that the 2D M-I-T *always* occurs in a regime dominated by random long range charged impurity scattering (and not by the short range interface roughness scattering, which is effective for $n_s > n_{max}$ and $\gg n_c$), and therefore screening is important. For low electron density and charged-impurity scattering-dominated transport the detailed temperature dependence of the mobility can be theoretically calculated by combining the Boltzmann equation with the dielectric screening formalism including finite temperature and level broadening effects, due to the charged impurities. Results of such calculations are in reasonable qualitative agreement with the experimental observations on the temperature and density dependence of the measured mobility on the metallic side ($n_s > n_c$) of the M-I-T. We define the conductivity $\sigma = ne\mu$ and the resistivity $\rho = 1/\sigma$, following the standard convention, and assume

that n_s is fixed by the gate voltage with all the T -dependence of σ or ρ arising from $\mu(T)$ *i.e.* the scattering rate. The calculation of ρ involves an energy averaging, which contributes quite significantly at higher temperatures, because the effective degeneracy or Fermi-temperature T_F of the system is quite low at low values of the density. In particular for $n_s = 10^{11}\text{ cm}^{-2}$ we have $T_F = 7\text{ K}$, with $T_F \propto n_s$ in 2D systems. The effective T_F may in fact be much lower because only the unbound electrons contribute to the Fermi degeneracy. The asymptotic $\rho(T)$ arising from charged impurity scattering at effective low $T < T_F$ and high temperatures $T > T_F$ can be calculated and one gets: $\rho(T) \sim \rho_0 + AT \equiv \rho_q$ for $T < T_F$ and $\rho(T) \sim BT^{-1} \equiv \rho_c$ for $T > T_F$. In calculating these asymptotic forms, we take into account only the screened charged impurity scattering, and all the temperature dependence arises from thermal smearing of the Fermi distribution function in screening and thermal averaging. All other sources of temperature dependence, such as phonons, weak localization, and interaction corrections to scattering have been neglected as quantitatively unimportant for Si MOSFETs in the temperature range ($T \sim 0.1 - 10\text{ K}$) of interest. The coefficients A and B arise from screening and energy-averaging corrections, respectively. The asymptotic low (ρ_q) and high (ρ_c) temperature resistivities given above correspond to the quantum ($T \ll T_F$) and the classical ($T \gg T_F$) carrier diffusion transport regimes respectively. The truly amazing thing to note about Si MOSFETs exhibiting a 2D M-I-T is that n_c is so low, $\sim 10^{11}\text{ cm}^{-2}$, and hence $T_F < \sim 7\text{ K}$ that the 'metallic' system just above the M-I-T shows a classical to quantum transport crossover in the narrow temperature range of $T \sim 0.1 - 10\text{ K}$. This transport crossover is reflected in the observed^{1,4} strong non-monotonicity of $\rho(T)$ on the metallic side of the transition ($n_s > n_c$), where $\rho(T)$ invariably rises with temperature for higher temperatures (as in ρ_c) before exhibiting the metallic behavior of a positive temperature coefficient (as in ρ_q) at lower temperatures where $\rho(T)$ decreases with lowering temperatures.

Detailed calculations show that at very low temperatures, when $T \ll T_D$ with $T_D \sim \Gamma/k_B$ as the Dingle temperature of the system and Γ the impurity scattering induced quantum level broadening, the coefficient $A \propto T$ and $\rho(T) \sim \rho_0 + O(T^2)$ at the lowest temperatures, becoming $\rho(T) \sim \rho_0 + O(T)$ for $T > T_D$. The impurity scattering limited $\rho(T)$ thus shows a crossover (for $n_s > n_c$) in its temperature dependence with $\rho(T)$ being 'metallic' for $T < T^*$ and 'insulating' for $T > T^*$ with $T^* \sim T_F/3$, being approximately the crossover temperature. Thus for higher densities T^* moves to higher temperatures, and the 'insulating' behavior on the metallic side gets progressively suppressed as phonons become more important. We emphasize that the temperature dependence of $\rho(T)$ arising from these effects could be quite large (a factor of 3 to 4 is usual on the metallic side, and a factor of 10 is quite possible) at these low densities.

The observed strong non-monotonicity in the experi-

mental $\rho(T)$ for $n_s \sim n_c$, which is naturally explained in our theory involving only charged impurity scattering, provides further qualitative and quantitative support for the FELPI scenario proposed in this article. Additional temperature dependence may also arise at low temperatures on the metallic side as the temperature becomes comparable to electronic binding energies (to ions), reducing screening dramatically.

Quantum phase transition?

The 2D M-I-T in Si MOSFETs and other 2D systems have been interpreted on the basis of a $T = 0$ quantum phase transition from a metallic phase (for $n_s \geq n_c$) to an insulating phase (for $n_s \leq n_c$) driven by the carrier density. The main support for the quantum phase transition scenario comes from the scaling collapse of $\rho(T, \delta n_s = |n_s - n_c|)$ data in Si MOSFETs around the transition point $n_s = n_c$ at low temperatures¹⁻³. This interpretation of the 2D M-I-T as a quantum phase transition has understandably attracted a great deal of theoretical attention⁶, particularly because the non-interacting non-linear sigma model based weak localization or single parameter scaling theories⁸ rule out such a phase transition in 2D as all states are at least weakly localized (in orthogonal and unitary ensembles – we will ignore the symplectic case relevant for spin-orbit scattering disorder), and any 2D M-I-T, in this widely accepted standard theory, can only be a crossover from a weak localization to a strong localization regime as n_s decreases. We discuss the quantum phase transition aspects of the 2D M-I-T in light of our proposed FELPI-scenario in this section.

In our proposed scenario the important issue we address is the existence of the sharp density n_c separating apparent strongly metallic ($n_s > n_c$) and strongly insulating ($n_s < n_c$) low temperature $\rho(T)$ behavior. We provide a specific microscopic mechanism (FELPI) where the fact that the 2D M-I-T always occurs in a situation involving few electrons per random charged impurity ion center plays a crucial role: for $n_s < n_c$, all the electrons are tightly bound or trapped in pairs to charged ions at the interface (without any 'free' electrons available for metallic transport), creating a strongly insulating state where only activated or variable range hopping transport is possible at low temperatures; for $n_s > n_c$, there are free electrons weakly scattering from negative U-centers³¹ created by charged ions with electron pairs bound to them, creating a strongly metallic state at low temperatures because scattering is substantially reduced by the bound electron pairs screening the ion. We can qualitatively explain the strong non-monotonic $\rho(T)$ on the metallic side ($n_s > n_c$) as an interplay of screening and thermal averaging as the system makes a quantum to classical transport crossover for $T \sim 0.1 - 10 K$ with $T_F \sim 5 K$. We can also explain the strong magnetic field

dependence based on the drastic decrease of screening on the metallic side and a strong increase in the binding energy on the insulating side due to the singlet-triplet transition. Electric field effects are easily explained in our scenario as field-induced stripping or tunneling of electrons due to their relatively small binding energies. Crucial factors in our FELPI-scenario are the dominance of charged impurity scattering controlling the 2D M-I-T physics, the small ($\sim 0.1 - 0.5 meV$) electronic binding energies to charged ions at the interface, the temperature dependence of the electronic screening, the low values of the degeneracy temperature causing the quantum to classical crossover, and the nonlinear self-consistent interplay of scattering/binding/screening between electrons and ions. All of these factors arise because of the low electron density in the system, which also produces strong inter-electron Coulomb correlations further enhancing the strongly insulating state, which takes on the character of a very strongly disordered electron solid (where few electrons are pinned or bound to individual random charged ions) or equivalently a strongly pinned Wigner glass.

It should be obvious from the above summary of the FELPI-scenario that the proposed microscopic mechanism makes no reference to a quantum phase transition, and in fact, based on our currently existing analysis of the problem, we cannot rule out a rapid crossover at $n_s \sim n_c$ from a metallic-like phase (for $n_s > n_c$) to a strong insulator phase (for $n_s < n_c$) as the number of electrons per ion in our scenario decreases from being above 2 (or, some other such small number) in the metallic phase to being below 2 in the insulating phase – by definition such a crossover in our FELPI-scenario will occur at a sharp electron density $n_s = n_c$ because the "M-I-T" takes place precisely as the number of electrons per charged ion passes through a fixed small number (taken to be 2 for our discussion, but it could be 4 with no loss of generality). Within our scenario the low density ($n_s < n_c$) phase is a strongly localized insulator by definition because all the carriers are strongly bound or trapped at random charged ion centers. The 'metallic' phase ($n_s > n_c$) in our scenario, however, may very well be a weakly localized (rather than truly extended) phase at $T = 0$ – our theoretical mechanism is completely insensitive to the eventual $T = 0$ phase of the 'metallic' electrons for $n_s > n_c$. What is important is that the system behaves as an 'effective metal' at the lowest temperatures typically attainable ($T > 0.1 K$) in the 2D M-I-T experiments the crossover to weakly localized behavior could occur at substantially lower temperatures; in addition the temperature dependence arising from screening, as considered in our work, is roughly two orders of magnitude stronger than weak localization induced temperature corrections in the experimentally relevant $0.1 - 10 K$ range, again making the $T = 0$ fate of the metallic phase inconsequential for understanding the M-I-T experiments.

While our FELPI scenario is a quantum scenario (be-

cause it involves quantum binding of few electrons per ion at low temperatures) we do not require any quantum phase transition to qualitatively understand the 2D M-I-T because our microscopic mechanism specifically incorporates a sharp density ($n_s = n_c$) at which the system changes from an 'effective metal' to a strong insulator. The transition is obviously continuous in our picture because it is induced by a continuous change in the electron density. We cannot rule out a quantum phase transition within our model, and whether there is a quantum phase transition driving the 2D M-I-T or not is beyond the scope of our paper, and is in fact quite irrelevant, in our opinion, to understanding the experimental data because the data are consistent with the metallic phase being a weakly localized system where the weak localization physics would only manifest itself at experimentally inaccessible low temperatures.

In this context it is important to point out that while the 2D M-I-T itself seems to be a generic phenomenon in the sense that the basic transition has now been observed in several different classes of 2D systems, the scaling collapse of the $\rho(T, \delta n_s = |n_s - n_c|)$ data has only been reported for Si MOSFETs. Even in the Si-MOSFETs, where the scaling behavior of the 2D M-I-T has been reported, one can legitimately question the empirical details of the scaling collapse in the sense that $\rho(T, n_s > n_c)$ shows considerable non-monotonicity as a function of temperature in the $T = 0.1 - 4 K$ range (which is naturally explained in our theory as described above) and therefore, by definition, it is impossible to have true scaling behavior except in a very small temperature and density window, where the concept of scaling becomes not particularly meaningful. Our view is that the essence of the 2D M-I-T can be understood on the basis of the FELPI scenario without invoking or requiring any quantum phase transition, and the existence (or not) of a true quantum phase transition in the 2D M-I-T phenomenon is open.

2 dimensional hole gas

Because of the experimental similarity the same physics must play a role in other reported 2D M-I-T *e.g.* two-dimensional hole gases. We argue that the FELPI-scenario plays a role in these systems too. The high mobility obtained in MBE-grown samples is a result of eliminating interface roughness and eliminating ionized donors in the conducting plane. Scattering is limited to the so-called remote ionized impurities. In the sample studied by Hanein et al⁴ the active layer is in a 150 nm thick quantum well capped by a p-doped GaAs top layer. Unfortunately the density of dopants is not given. It is clear though that the range over which the carrier density is varied runs from $0.089 \cdot 10^{11} \text{ cm}^{-2}$ to $0.64 \cdot 10^{11} \text{ cm}^{-2}$ with a critical value located somewhere around $0.25 \cdot 10^{11} \text{ cm}^{-2}$. If we would assume that this is

also at roughly 3 holes per ion, the total range is from 1 to 7 holes per ion, which is consistent with the FELPI scenario.

In the sample studied by Simmons et al⁵ a 20 nm thick quantum well is used and the nearby GaAs layer is Si modulation doped. Also in this case the density of dopants is not mentioned. In this experiment one finds a critical value of $n_c \sim 0.5 \cdot 10^{10} \text{ cm}^{-2}$. If we again assume that the critical value is at about 3 holes per ion in this experiment the total change occurs in a range from $0.3 - 0.7 \cdot 10^{11} \text{ cm}^{-2}$ or from 2 to 4 holes per ion.

While the actual numbers and the details are not as well known in the other systems as in Si MOSFETs, we can, however, assert that the FELPI scenario predicts a much lower value for n_c because the charged impurities are further away from the electrons (holes). The same non-monotonicity in $\rho(T)$, as discussed here, is seen⁴ in these GaAs systems also. In addition the recently observed³³ re-entrant insulator-metal transition at higher hole-densities might easily fit into our qualitative picture as evidencing the onset of binding of holes to the ions, and the re-entrance also is evidence in favor of crossover physics.

Conclusions

We have proposed a theoretical scenario, based on the few-electrons-per-ion (FELPI) mechanism, for the observed 2D M-I-T in Si MOSFETs. Our theoretical scenario, in contrast to most other proposed theories of the phenomenon, makes extensive use of the actual experimental parameter values operating in the phenomenon – in particular we believe that the crucial process is the dominance of charged impurity scattering in the low density systems, which controls the physics of the 2D M-I-T. We believe that in the insulating phase ($n_s < n_c$) all the electrons are bound in pairs (or, some other small numbers) producing an insulating state whereas in the metallic phase ($n_s > n_c$), there are excess free electrons, which scatter from ions with bound pairs (or, some other small number) of electrons. We can qualitatively explain understand the density/temperature/magnetic field/electric field dependence of the observed resistivity both on the metallic and the insulating sides of the transition using the FELPI scenario, and also the fact that, at the low electron densities involved in the problem, the 2D M-I-T is experimentally close to a quantum-classical crossover regime. Our theory naturally provides an explanation for why $n_c \sim 10^{11} \text{ cm}^{-2}$ in Si MOSFETs – it is simply related to the ionic charge density being around a few times 10^{10} cm^{-2} , provided high-mobility MOSFETs are used and the electron-electron interaction dominates over the kinetic energy part of the electrons. Quantum phase transition considerations, which have created considerable excitement in the literature, turn out to be irrelevant to (and beyond the scope of) our proposed

mechanism. Our theory makes precise predictions and is falsifiable – for example, any experimental observation of a 2D M-I-T in Si-MOSFETs in the surface roughness scattering dominated regime *i.e.* $n_c > n_{max}$ will prove our theory wrong. Another example is our prediction that any lowering of the random ionic charge density will automatically push n_c lower. While many theoretical details of our proposed FELPI scenario remain to be worked out, our proposal has the merit of being precise and concrete, which should enable one to calculate its narrow consequences to test the quantitative validity of FELPI for the 2D M-I-T problem.

Acknowledgments

We like to thank S.V. Kravchenko for stimulating discussions and correspondence about the experimental results. Our work is supported by the NSF (both at QUEST and ITP, UCSB and at UMCP) and the ONR (SDS). In addition TMK likes to thank the Netherlands-America Committee for Educational Exchange for a Fulbright grant to partially support his stay at UCSB.

-
- ¹ S.V.Kravchenko, G.V.Kravchenko, J.E.Furneaux, V.M.Pudalov, and M.D'Iorio, Possible metal-insulator transition at $B = 0$ in two dimensions, *Phys. Rev.***B50**, 8039-8042(1994)
 - ² S.V.Kravchenko, W.E.Mason, G.E.Bowker, J.E.Furneaux, V.M.Pudalov, and M.D'Iorio, Scaling of an anomalous metal-insulator transition in a two-dimensional system in silicon at $B = 0$, *Phys. Rev.***B51**, 7038(1995)
 - ³ S.V.Kravchenko, D.Simonian, M.P.Sarachik, W.Mason, and J.E.Furneaux, Electric field scaling at a $B = 0$ metal-insulator transition in two dimensions, *Phys.Rev.Lett.* **77**, 4938 (1996)
 - ⁴ Y.Hanein, U.Meirav, D.Shahar, C.C.Li, D.C.Tsui, and H.Shtrikman, The metalliclike conductivity in a two-dimensional hole system, *Phys. Rev. Lett.*, **80**,1288-1291(1998)
 - ⁵ M.Y.Simmons, A.R.Hamilton, M.Pepper, E.H.Linfeld, P.D.Rose, D.A.Ritchie, A.K.Savchenko, and T.G.Griffiths, Metal-insulator transition at $B = 0$ in a dilute two dimensional GaAs-AlGaAs hole gas, *Phys. Rev. Lett.* **80**,1292-1295(1998)
 - ⁶ B.Goss Levy, Metal-insulator transition unexpectedly appears in a two-dimensional electron system, *Phys. Today* **50**,19(1997)
 - ⁷ E.Abrahams, P.W.Anderson, D.C.Licciardello, and T.V.Ramakrishnan, Scaling theory of localization: absence of quantum diffusion in two dimensions, *Phys. Rev. Lett.* **42**, 673-676 (1979)
 - ⁸ V.Dobrosavljević, E.Abrahams, E.Miranda, and S.Chakravarty, Scaling theory of two-dimensional metal-insulator transitions, *Phys. Rev. Lett.* **79**, 455 (1997)
 - ⁹ S.L.Sondhi, S.M.Girvin, J.P.Carini, and D.Shahar, Continuous quantum phase transitions, *Rev. Mod.Phys.***69**, 315 (1997)
 - ¹⁰ C.Castellani, C.Di Castro, and P.A.Lee, Metallic phase and metal-insulator transition in two-dimensional electronic systems, *Phys.Rev.***B57**, R938 (1998)
 - ¹¹ Philip Phillips, Yi Wan, Ivar Martin, Sergey Knysh, and Denis Dalidovich, Superconductivity in a two-dimensional electron gas, *Nature* **395**,253-257(1998)
 - ¹² D.Belitz and T.R.Kirkpatrick, Possible triplet superconductivity in MOSFETs, *Phys.Rev.***B58**,8214-8217(1998)
 - ¹³ S.He and X.C.Xie, New liquid phase and metal-insulator transition in Si MOSFETs, *Phys. Rev.Letters***80**, 3324(1998)
 - ¹⁴ S.Chakravarty, S.Kivelson, C.Nayak and K.Völker, Wigner Glass, Spin Liquids and the Metal-Insulator Transition, *cond-mat/9805383 v2* 22 Aug. 1998
 - ¹⁵ Qimiao Si and C.M.Varma, Metal-insulator transition of disordered interacting electrons, *Phys. Rev. Lett.***81**, 4951-4954 (1998)
 - ¹⁶ B.L.Altshuler and D.L.Maslov, On the theory of metal-insulator transitions in gated semiconductors, *Phys.Rev.Lett.***82**, 145-148 (1999)
 - ¹⁷ T.Ando, A.B.Fowler, F.Stern, Electronic properties of two-dimensional systems, *Rev. Mod.Phys.* **54**, 437-672(1982)
 - ¹⁸ G.H.Kruithof, T.M.Klapwijk and S.Bakker, Temperature and interface-roughness dependence of the electron mobility in high-mobility Si(100) inversion layers below 4.2 K, *Phys.Rev.* **B43**, 6642-6649(1991)
 - ¹⁹ G.H.Kruithof and T.M.Klapwijk, Folklore and science in High-mobility MOSFETs, in *Condensed Matter of Low Dimensionality* edited by J.L.Beeby *et al*, Plenum Press, New York, 1991
 - ²⁰ R.Heemskerk and T.M.Klapwijk, Nonlinear resistivity at the metal-insulator transition in a two-dimensional electron gas, *Phys. Rev.***B58**,R7154-R7157(1998)
 - ²¹ V.M.Pudalov, M.D'Iorio, S.V.Kravchenko, and J.W.Campbell, Zero-magnetic field collective insulator phase in a dilute 2D electron system, *Phys. Rev. Letters*,**70**, 1866-1869(1993)
 - ²² E.A.Vyrodov, V.T.Dolgoplov, C.I.Dorozhkin, and N.B.Zhitenev, Temperature dependence of conductivity in high mobility MIS structures on a base of (001) silicon, *Zh.Eksp.Teor.Fiz.***94**,234(1987) [*Sov.Phys.JETP* **67**,998(1988)]
 - ²³ Y.Hanein, D.Shahar, J.Yoon, C.C.Li, D.C.Tsui, and H.Strikhman, Observation of the Metal-Insulator-Transition in Two-dimensional n-type GaAs, *cond-mat/9808251* 24 August 1998
 - ²⁴ D.Popović, A.B.Fowler, and S.Washburn, Metal-insulator transition in two dimensions: effects of disorder and magnetic field, *Phys. Rev. Lett.* **79**, 1543-1546 (1997)
 - ²⁵ D.Simonian, S.V.Kravchenko, M.P.Sarachik and V.M.Pudalov, Magnetic field suppression of the conducting phase in two dimensions, *Phys. Rev.Letters***79**,2304-2307(1997)
 - ²⁶ S.V.Kravchenko, private communications
 - ²⁷ B.Vinter, Influence of charged impurities in Si inversion-

- layers, *Phys. Rev.* **B26**,6808(1982)
- ²⁸ S.T.Chui and B.Tanatar, Phase-diagram of the two-dimensional quantum electron freezing with external impurities, *Phys.Rev.*,**B55** 9330-9332 (1997)
- ²⁹ Jongsoo Yoon, C.C.Li, D.Shahar, D.C.Tsui, and M.Shayegan, Wigner crystallization and metal-insulator transition of two-dimensional holes in GaAs at $B = 0$, *cond-mat/9807235 v2* 22 Jul. 1998
- ³⁰ I.K.Marmorkos, V.A.Schweigert, and F.M.Peeters, Remote and spatially separated D^- centers in quasi-two-dimensional semiconductor structures, *Physical Review*,**55**, 5065-5072(1997)
- ³¹ P.W.Anderson, Model for the electronic structure of amorphous semiconductors, *Phys.Rev.Letters* **34**,953-955(1975)
- ³² S. Das Sarma and E.H.Hwang, Charged impurity scattering limited low temperature resistivity of low density silicon inversion layers, *cond-mat/9812216* 14 Dec. 1998
- ³³ A.R.Hamilton, M.Y. Simmons, M.Pepper, E.H.Linfield, P.D.Rose, and D.A.Ritchie, Re-entrant insulator-metal-insulator transition at $B = 0$ in a two dimensional hole gas, *cond-mat/9808108*, 11 Aug. 1998